

## GhY][YbXY'5i hcj Yfg]WXYfi b[gVY]hfÀ[YZØf'6Ui hnYb'i bX'GÀWXg]gWXY'GWXkY]n

 $: \varnothing f'j ]Y`Y : U\fYf']b `GUW gYb `ghY][ Yb `X]Y ? cghYb `XYf ? Zn! < UZhd Z]W hj Yfg]W Yfi b[ "`BYi YghY `FY[ ]cbU`_`UggYb `_ÒbbhYb ni `\Ò\YfYb `6Y]hfÀ[ Yb `Z\varnothing\fYb"$ 

± GUW gYb ghY\Yb J YfÀb XYfi b [ Yb ]b XYf 5i hc!J Yfg]W Yfi b [ `Ubž X]Y ZØf Y]b Y J ]Y `nU\` 'j cb : U\ fYfb Yf\ YV`]W Y Z]b Ubn]Y ``Y

5i gk ]f\_i b [ Yb `\ UVYb `\_ ObbhY b "`Fi b X '' \$\$ "\$\$\$ 5i hc ZU\ fYf ]b `XYb

@Ub X\_fY]gYb `6Ui hnYb i b X `GÀW g]gW Y `GW k Y]n `a ØggYb `\_ Øb Zh][

a ]h `\ O\ YfYb `6Y]hfÀ [ Yb `]b `XYf `? Zn! < UZhd Z]W hj Yfg]W Yfi b [

fYW b Yb " 8 Yf ; fi b X `X UZØf ]gh Y]b Y `gW `Y W hY fY `9]b ghi Zi b [ `]b `X]Y

F Y [ ]c b U`\_ `UggY ž X]Y `X]fY\_h `a ]h `XYa `F ]g]\_c 'j c b `GW ÀXYb `]b `X]YgYb

; YV]YhYb `ni gUa a Yb\Àb [ h"

8 Yf; YgUa hj YfVUbX:XYf:8 Yi hgWXYb:J Yfg]WXYfi b[gk]fhgWXUZh fl 8J &:\Uh:X]Y:'\A\f`]WXY:GWXUXYbgV]`Ubn:VYfYWXbYhž:X]Y:'Ui Z:XYb Ni `Uggi b[gVYn]f\_Yb:]b:8 Yi hgWX:\UbX:\VUg]Yfh":8]YgY:'9]bghi Zi b[:\Uh YbhgWXY]XYbXY:5i gk]f\_i b[Yb:'Ui Z:X]Y:?cghYb:'XYf 5i hcj Yfg]WXYfi b[":: Øf:X]Y:\ØVf][Yb:'Y`Z:Ni `Uggi b[gVYn]f\_Y:]b GUWXgYb:'V`Y]Vh:X]Y:'9]bghi Zi b[:'YXcWX:i bj Yf\AbXYfhz:k Ug:'Z\Øf:j]Y`Y 5i hcZU\fYf:Y]bY:'9bh`Ughi b[:'XUfghY``h"

## ŸbXYfib[Yb]bXYf?Ug\_cjYfg]WXYfib[

 $B ] WX h b i f X] Y < UZhd Z ] WX h j Yfg] WX Yfi b [ ']gh 'VY h f c ZZY b / `Ui WX 'VY] 'XY b ? Ug_c j Yfg] WX Yfi b [ Yb '[ ]Vh 'Yg 'VY XY i h Y b XY '5 b d Uggi b [ Yb '' G c d f c Z]h] Y f Y b 'b Y i b 'j c b 'X f Y]n Y \ b 'Ni `Uggi b [ g V Y n] f_Y b 'V Y] 'X Y f H Y]`_Ug_c 'j c b 'Y] b Y f 'V Y g g Y f Y b '9] b g h Zi b [ ž' k À \ f Y b X '] b 'G h A X h Y b k ] Y '8 f Y g X Y b 'i b X '@ Y] d n] [ 'g c k ] Y '] a '6 Y n] f_ 'G A WX g] g WX Y 'G WX k Y] n! C g h Y f n[ Y V] f [ Y 'U `` Y g 'V Y] a '5 `h Y b 'V `Y] V h '' 6 Y] 'X Y f 'J c `` _Ug_c 'k ] f X 'X] Y$ 

9]bghi Zi b[ 'ZØf'; Òf`]hnž'A]hhY`gUW\gYb'i bX'Nk] $W_U$ i '[ Øbgh][ Yfž'k Ug]bg[ YgUa h'ZØf'Y]b][ Y'5i hcZU\fYf'Y]bY'dcg]h]j Y'9bhk] $W_L$ `i b[ XUfghY``h"

 $8 ]Y \cdot 9]bghi Zi b[Yb \cdot \setminus UVYb \cdot X]fY_hY \cdot 5i gk]f_i b[Yb \cdot XUfUi Zž \cdot k]Y \cdot j]Y \cdot 5i hc ZU \setminus fYf \cdot ZØf \cdot j \setminus fY \cdot J Yfg]W \setminus Yfi b[Yb \cdot VYnU \setminus Yb \cdot a ØggYb'' \cdot X' VYggYf X]Y \cdot FY[]cbU \cdot _ ` UggY z \cdot XYghc \cdot [Yf]b[Yf \cdot XYf \cdot 6Y]hfU[z \cdot k Ug \cdot ZØf \cdot j ]Y \cdot Y J Yfg]W \setminus YfhY \cdot ]b \cdot GUW \cdot gYb \cdot j \cdot cb \cdot 6YXYi hi b[ \cdot ]gh'' \cdot \dangle X]YgYa Ni gUa a Yb \ Ub[ \cdot ]gh']bh \ fYggUbh \ ni \cdot k \ ]ggYb z \ XUgg \ XYf \ ; 8 J \ a \]h gY]b \ b \ 6YfYW \ bi b[Yb \ XUfUi Z \ UVn]Y \ ` hz \ X]Y \ 7 \ Ub \ W b \ ` Ui Z \ GW \ A XYb \]b \ VYgh]a a \ hYb \ [Yc[fUZ]gW \ Yb \ FY[]cb Yb \ ` UVni V] \ XYb \ "8 \ Ug \ VYXYi \ hYhz \ XUgg \ ]b \ FY[]cb Yb \ a \ ]h'b] \ YXf][YfYb \ GW \ UXYbgnU \ ` Yb \ X]Y \ J \ Yfg]W \ Yfi \ b[g_cghYb \ hYb XYbn]Y \ ` ` b] YXf][Yf \ g]b X''$ 

 $\pm 0.'XYf'?Zn! < UZhdZ]WXhj Yfg]WXYfi b[ '[ Y`hYb']bg[ YgUa h`nk Ò`Z FY[ ]cbU`_`UggYb''' < ÒWXghY'9]bghi Zi b[ Yb']b'GUWXgYb'Z]bXYb'g]WX']b XYb'GhÀXhYb'8fYgXYb'i bX'@Y]dn][ ž'k c'X]Y'GWXUXYbgV]`Ubn'hYi fYf ]ghž'k À\fYbX'; Òf`]hn'i bX'A ]hhY`gUWXgYb'X]Y'[ Øbgh][ ghYb'FY[ ]cbYb XUfghY``Yb''' < ]Yf'dfcZ]h]YfYb'5i hcZU\fYf'j cb'[ Yf]b[ YfYb'6Y]hfÀ[ Yb''$ 

#### $8]Y 6YXYi hi b[X]YgYf 9bhk]W_i b[Yb$

K Ufi a 'g]bX'X]YgY'bYi Yb'9]bghi Zi b[Yb'j cb'gc'W\'[fc\Yf 6YXYi hi b[3'G]Y'V]YhYb'b]W\h'bi f'Y]bYb'9]bV']W\_']b'X]Y
F]g]\_c`UbXgW\UZh'XYg'5i hc ZU\fYbg']b'GUW\gYbž'gcbXYfb'nY][YbUi W\X]Y'Z]bUbn]Y'`Yb'6Y`Ughi b[Ybž'a]h'XYbYb': U\fYf']b
VYgh]a a hYb'FY[]cbYb'\_cbZfcbh]Yfh'k YfXYb''8]YgY'FY[Y'i b[YbjYf\bXYfb'X]Y'Z]bUbn]Y'`Y'D`Ubi b['j]Y`Yf'A YbgW\Ybž'X]Y'Ui Z']\fY5i hcg'Ub[Yk]YgYb'g]bXž'i bX'fYZ'Y\_h]YfYb'VfY]hYfY'HfYbXg
]bbYf\U'V'XYf'JYfq]W\Yfi b[gk]fhgW\UZh''

8 UfØVYf`\]bUi g`cZZYbVUfh`XUg`GmghYa `XYf`FY[]cbU`\_`UggYbž`XUgg F]g]\_cVYk Yfhi b[ 'b]Wkh'bi f`Y]b`]bX]j ]Xi Y``Ygž`gcbXYfb`Ui Wk`Y]b fY[]cbU`Yg`D\Àbca Yb`]gh" \dangle 'Y]bYf`NY]hž`]b`XYf`X]Y 5i hcj Yfg]WkYfi b[ 'gck c\``ZØf`Df]j UhdYfgcbYb`U`g`Ui Wk`ZØf : Ua ]`]Yb`ni 'Y]bYa 'k YgYbh`]WkYb`?cghYbZU\_hcf`k ]fXž`g]bX`gc`WkY \dangle Zcfa Uh]cbYb`ZØf`U``Y`6YhY]`][ hYb`j cb`[ fc»Yf`FY`Yj Ubn" 9gʻ]ghʻYXcW\`k]W\h][`ni`VYhcbYbž'XUggʻY]bYʻJYfÀbXYfi b[`XYfFY[]cbU`\_`UggYʻb]W\hii bVYX]b[hʻ5i ggU[YbʻØVYfʻXYbʻ[YgUa hYb?Zn!JYfg]W\Yfi b[gVY]hfU[`ni`Àggh"8]Yʻj]Y`ZÀ`h][Ybʻ: U\_hcfYbž'X]YX]Y`JYfg]W\Yfi b[gVY]hfÀ[Y`VYY]bZi ggYbžʻg]bX`kY]hfY]W\YbX`i bX\_ca d`YI"

#D'X]YgYa '?cbhYl h'`Àggh'g]W\'Ui W\'ZYghghY``Ybž'XUgg'VYghY GW\UXYbgV]`UbnYb']b'UbXYfYb'HY]`Yb'8 Yi hgW\`UbXgž'k ]Y']a '6Yn]f\_ 9`VY!9`ghYf']b'6fUbXYbVi f[ž'k c'a Ub']b'X]Y'VYghY'FY[]cbU`\_`UggY fP`UggY'%L'Y]b[Yghi Zh']ghž'k Y]hYf\]b'A Ò[`]W\\_Y]hYb'ZØf'dcg]h]j Y 9bhk]W\_`i b[Yb'ZØf'5i hcZU\fYf'VYXYi hYb'\_ObbYb"'8]YgY'8 UhYb k Y]gYb'XUfUi Z'\]bž'XUgg'Yg'FY[]cbYb'[]Vhž']b'XYbYb'5i hcZU\fYb XYi h`]W\'a ]h'k Yb][Yf'GW\UXYbgZÀ``Yb'j YfVi bXYb']gh"

### 5 i gV`]W\_'Ui Z'X]Y'\_ØbZh][Y'9bhk ]W\_`i b[

5b[Yg]Wkhg'XYf'ni bY\a YbXYb'?ca d`Yl]hÀh'XYf

J Yfg]WkYfi b[gg]hi Uh]cb']gh'Yg'ZØf'X]Y'5i hcZU\fYf'g]bbj c``ž'g]Wk

ØVYf']\fY'YkY]`][Yb'9]bghi Zi b[Yb'i bX'a Ò[`]WkY'JYfÀbXYfi b[Yb
ni ']bZcfa ]YfYb"'Jcf'U``Ya ']b'NY]hYbž']b'XYbYb'g]Wk'X]Y

F U\a YbVYX]b[i b[Yb'gWkbY``'j YfÀbXYfb'\_ÒbbYbž']gh'Yg'k]Wkh][ž
[i h']bZcfa ]Yfh'ni 'gY]bž'i a 'X]Y'VYghYb'9bhgWkY]Xi b[Yb'ZØf'X]Y

Y][YbY'Z]bUbn]Y``Y'Ni \_i bZh'ni 'hfYZZYb"

# < ]bhYf[ fi bX]bZcfa Uh]cbYb ni f ? Zn! J Yfg]WX Yfi b[ ]b 8 Yi hgWX `UbX</pre>

8]Y?Zn!JYfg]WXYfi b[']gh']b'8Yi hgWX`UbX'[YgYhn`]WX
j cf[YgWXf]YVYb"'\*XYYf': U\fnYi [\U`hYf'a i gg'Y]bY
<UZhdZ]WXhj Yfg]WXYfi b['UVgWX`]Y\*Ybži a 'X]Y'Xi fWX'XUg': U\fnYi [
j Yfi fgUWXhYb'GWXÀXYb'Ub'8f]hhYb'UVni g]WXYfb"'5V[YgY\Yb'j cb
XYf'<UZhdZ]WXhj Yfg]WXYfi b['k À\`Yb'j ]Y`Y'5i hcZU\fYf'ni XYa 'Y]bY
HY]`!'cXYf'Jc``\_Ug\_cj Yfg]WXYfi b[ži a 'Ui WX'Y][YbY'GWXÀXYb
UVni XYW\_Yb"

8]Y'DfÀa]Yb'ZØf'?Zn!JYfg]W\Yfib[Yb'g]bX']b'8Yi hgW\`UbX'j cbjYfgW\]YXYbYb': U\_hcfYb'UV\Àb[][ž'XUfibhYf'X]Y'FY[]cbž'XUg

 $: U\fnYi\ [\ \check{z}'XUg':\ U\fYfVU`hYb'i\ bX'XYf'dYfgOb`]WXY\\ GWXUXYbj\ Yf`Ui\ Z'XYg':\ U\fYfg"'8Yf';\ YgUa\ hj\ YfVUbX'XYf'8Yi\ hgWXYb\ J\Yfg]WXYfi\ b[\ gk\ ]fhgWXUZh'fl\ 8J Ł'gd]Y`h'Y]bY'nYbhfU`Y'Fc``Y'VY]'XYf\ 6YfYWXbi\ b[\ XYf'FY[\ ]cbU`_`UggYbž'X]Y'XUg'F]g]_c'j\ cb'GWXAXYb']b\ j\ YfgWX]YXYbYb';\ YV]YhYb'k\ ]XYfgd]Y[\ Y`b"'ŸbXYfi\ b[\ Yb']b'X]YgYb\ ?`UggYb'\UVYb'X]fY_hY'5i\ gk\ ]f_i\ b[\ Yb'Ui\ Z'X]Y\ J\ Yfg]WXYfi\ b[\ gVY]hfA[\ Y'XYf'5i\ hc\ ZU\ fYf''$ 

## 8 Yf '9]b Z'i gg'j cb'fY[]cbU'Yb': U\_hcfYb'Ui Z'X]Y J Yfg]WX Yfi b[g\_cghYb

FY[]cbU`Y`I bhYfgW\]YXY`]b`XYf`GW\UXYbghUh]gh]\_`VYY]bZi ggYb`X]Y
9]bghi Zi b[`]b`X]Y`FY[]cbU`\_`UggYb": U\_hcfYb`k ]Y`X]Y`8]W\hY`XYg
JYf\_Y\fgž'X]Y`5bnU\``XYf`I bZÀ``Y`i bX`gc[Uf`X]Y`?f]a ]bU`]hÀhgfUhY
]b`VYgh]a a hYb`; YV]YhYb`\_ÒbbYb`XUni `ZØ\fYbž'XUgg`X]Y`DfÀa ]Yb
ghY][Yb`cXYf`ZU``Yb"`Jcf`U``Ya `]b`ghÀXh]gW\Yb`; YV]YhYbž']b`XYbYb
XUg`JYf\_Y\fgUi Z\_ca a Yb`\cW\`]ghž'g]bX`X]Y`DfÀa ]Yb`hYbXYbn]Y``
\O\Yf`U`g`]b```ÀbX`]W\YfYb`FY[]cbYb"

±0 'X]YgYa 'Ni gUa a Yb\Ub[ ']gh'Yg'k ]W\h][ 'ni 'VYUW\hYbž'XUgg'Ui W\gcn]cÒ\_cbca ]gW\Y': U\_hcfYb'k ]Y'XUg'8 i fW\gW\b]hhgY]b\_ca a Yb']b XYf'FY[ ]cb'i bX'X]Y'U``[ Ya Y]bY'k ]fhgW\UZh`]W\Y'@U[ Y'Y]bY'Fc``Y gd]Y`Yb'\_ObbYb'''Gc'\_ObbYb''; YV]YhY'a ]h'\O\YfYf'5fVY]hg`cg][ \_Y]h cXYf'b]YXf][ YfYb'9]b\_ca a Yb'UbZ\\b[`]W\Yf'\O\YfY JUf]UV`Yb'ghY``Yb'g]W\Yfž XUgg'XUg'Df]W\\b[ gVY]hf\\alpha[ Y'Ui Zk Y]gYb'''8]YgY'JUf]UV`Yb'ghY``Yb'g]W\Yfž XUgg'XUg'Df]W\\b[ ']b'XYf'? Zn!JYfg]W\Yfi b[ gVfUbW\Y']bX]j ]Xi Y``i bX fY[ ]cbU`'[ YfYW\hZYfh][ h']gh"

#### 5\_hi Y``Y'GhUh]gh]\_Yb'ni f'? Zn!J Yfg]WX Yfi b[

@Ui h'XYb''Øb[ghYb''9f\YVi b[Yb'XYg'; 8J''[]Vh'Yg'a ca YbhUb'Yhk U (, 'A]``]cbYb'? Zn!JYfg]W\Yfi b[Yb']b'8Yi hgW\`UbX"'=a '\\\f'\&\\$&\&VYhfi [Yb'X]Y''9]bbU\a Yb'Ui g'? Zn!JYfg]W\Yfi b[Yb']bg[YgUa h'fi bX &- 'A]``]UfXYb''9i fc"'8]Y'6fUbW\Y'j YfnY]W\bYh'Y]bY'ghYh][Y Ni bU\a Y'Ub'Cb`]bY!JYf['Y]W\gdcfhU`Ybž'k Ug'Yg'XYb JYfVfUi W\Yfb'Yf`Y]W\hYfhž'X]Y'VYghYb'HUf]ZY'ni 'Z]bXYb"

9]bY'l a  $ZfU[Y'XYg'; 8J'ibhYf'5ihcZU\fYfb'nY][hž'XUgg'[ih'+$]$ 

 $\label{lem:decomposition} DfcnYbh'XYf'6YZfU[hYb'VYfY]h'g]bXž'X]Y'JYfg]WXYfib[g[YgY``gWXUZhni'k YWXgY`bž'k Ybb'g]Y'Y]bYb'VYggYfYb'HUf]Z'Z]bXYb'''8]Yg'nY][hXUg'k UWXgYbXY'\DhYfYggY'XYf'JYfVfUi WXYf'Ub'?cghYbYZZ]n]YbnibX'HfUbgdUfYbn']b'XYf'JYfg]WXYfib[gVfUbWXY'''$ 

8 UfØVYf`\]bUi g`]gh`XYf`HfYbX`ni `i a k Y`hZfYi bX`]W\YfYb
: U\fnYi [Yb`i bX`U`hYfbUh]j Yb`5bhf]YVYb`ni `VYcVUW\hYb"`@Ui h
Y]bYf`GhUh]gh]\_`XYg`?fUZhZU\fh!6i bXYgUa hYg`f?65Ł`g]bX`]a `\$J\f
&\$&&`ØVYf`%\$`DfcnYbh`XYf`bYi `ni [Y`UggYbYb`: U\fnYi [Y`9`Y\_hfc!
cXYf`<mVf]XUi hcg"`8]Yg`\_ÒbbhY`ni \_ØbZh][`9]bZi gg`Ui Z`X]Y
9bhk]W\_`i b[`j cb`gdYn]Y``Yb`HUf]ZYb`ZØf`i a k Y`hVYk i gghY
5i hcZU\fYf`\UVYb"

8 YhU]`q

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'X]Y!bUWX f]WX hYb "bYh