

6Yn]f_g`][]gh'<H6'jcf'Dc_U`!8iY``. HfU]bYf'\cZZh'UiZ'fYgdY_hUV`Yg'9f[YVb]g

<Ua Vi f[""+b 'Y]bYf'gdUbbYbXYb '9bhgWXY]Xi b[']a '@chhc!Dc_U`gd]Y`
XYf'Xf]hhYb 'Fi bXY'hf]ZZh'XYf' < UfVi f[Yf'H6 'Ui Z'XYb
C VYf`][Ua Y]ghYf'5`hcbU'- ' "'K À\fYbX'X]Y 'F c``Ybj YfhY]`i b['_`Uf'ni
gY]b 'gWXY]bhž'V`Y]Vh'X]Y': fU[Y ']a 'F Ui a ž'cV'XYf'6Yn]f_g`][]gh'Yg
gWX UZZYb '_Ubbž'XYb': Uj cf]hYb'ni 'ØVYffUgWXYb"</pre>

FYU`]gh]gWXY'9]bgWXÀhni b[Yb'XYf A UbbgWXUZh

8]YgY'G_Ydg]g'Yf]bbYfh'Ub'X]Y'G]hi Uh]cb'j cb'?`i V'?cgcj UžY]bYa@UbXYg`][]ghYbž'XYf']b'XYf'nk Y]hYb'Fi bXY'ØVYffUgWXYbX'[Y[YbXYb'FY[]cbU``][]ghYb': 7'HYi hcb]U'\$)'[Yk Ubbž'bUWXXYa']\f HfU]bYf'H\cfghYb'6YmYf'XUg'HYUa'YVYbZU``g'U`g'WXUbWYb`cgVYnY]WXbYh'\UhhY"'\J\`Ugg'gY`Vgh'g]Y\h'YXcWX'Y]bYb'gWXUfZYbIbhYfgWX]YX'nk]gWXYb'X]YgYb'VY]XYb'G]hi Uh]cbYb.' 8Ug'_Ubb'a Ubb]WXh'j Yf[`Y]WXYb"'5i WX'?cgcj U'k ØfXYb'k]f'U_hi Y``k c\`'b]WXhgWX`U[Yb" 'A]h'X]YgYf'Y\f`]WXYb'FYZYI]cb'a UWXh'Yf'X]Y'7\UbWYbgY]bYf'A UbbgWXUZh'XYi h`]WX"

9]b a O[`]WX Yf @]WX hV `]W_

5`hcbU'-' '\]b[Y[Yb'k]fX'[Yk Ufbh'gY]b'i bX']\fY'Y][YbYb
5a V]h]cbYb']a 'Dc_U`'b]W\h'Ui g'XYb'5i [Yb'j Yf']YfYb"'8Yf
Wi V\]ghcf]gW\Y'9fZc`[ž'[YdUUfh'a]h'XYf'9fk Ufhi b[g\U'hi b['XYf
: Ubgž'_ObbhY'8fi W_'YfnYi [Ybž'UVYf'Ui W\'X]Y'A ch]j Uh]cb'ghY][Yfb"
9]b'A UhW\'nk]gW\Yb'Y]bYa '6Yn]f_g`][]ghYb'i bX'Y]bYa
CVYf`][]ghYb'\Uh'cZh'XUg'DchYbn]U`'ZØf', VYffUgW\i b[Ybž'Ui W\
k Ybb'X]Y'GhUh]gh]_Yb'ghUf_'[Y[Yb'XYb'5i »YbgY]hYf'gdfYW\Yb"

8 Ugʻ\Yi h][YʻGd]Y``k]fX`b]W\h'bi fʻZØfʻX]YʻGd]Y`YfžʻgcbXYfbʻUi W\ʻZØf X]Yʻ: Ubgʻi bXʻXYbʻJ YfY]bʻY]bYʻ< YfUi gZcfXYfi b["'5i W\`k Ybb < UfVi f[YfʻH6ʻUʻgʻW\UbWʻb`cgʻVYhfUW\hYhik]fXžʻV`Y]VhʻX]Y HUhgUW\YžʻXUggʻ: i » VU``ʻcZhii bj cf\YfgY\VUfYʻK YbXi b[Ybʻb]a a h'' 8]YʻI bhYfghØhni b[ʻXYfʻNi gW\Ui Yfa UggYbʻ_Ubbʻ]bʻgc`W\Yb A ca YbhYbʻXYfʻYbhgW\Y]XYbXYʻ: i b_YbʻgY]bžʻXYfʻY]bʻGd]Y``_]ddYb `Àggh''

9]b 6`]W ni fØW i bX bUW j cfb

8]Y'G]hi Uh]cb'XYg'<UfVi f[Yf'H6G']a '@chhc!Dc_U`'_ObbhY'Y]bYb
K YbXYdi b_h']b'XYf'JYfY]bg[YgWk]WkhY'XUfghY``Ybž'i bUV\Ab[][
j ca '9f[YVb]g"'G]Y'hfYhYb'a]h'Y]bYa 'i bYfgWk@hhYf`]WkYb
HYUa [Y]gh'Ubž'i bX'Ui Wk'k Ybb'X]Y'9f[YVb]ggY'b]Wkh']a a Yf
ZUj cfUVY``Ui gZU``Ybž'V`Y]Vh'X]Y'@Y]XYbgWkUZh'ZØf'XUg'Gd]Y``i bX'X]Y
; Ya Y]bgWkUZh'i a 'XYb'JYfY]b'VYghY\Yb"'8]Y': À\][_Y]hž'[Y[Yb
ghÀf_YfY'; Y[bYf'ni '_Aa dZYb'i bX'g]Wk'k Y]hYfni Ybhk]\W_Y`bž
nY]WkbYh'Y]bY'A UbbgWkUZh'Ui g' 'i bX'[YbUi 'X]YgY'A YbhU`]hÀh
_ObbhY'XYf'GWk'\ØggY`'gY]bž'i a 'g]Wk']b'XYf'K Yhh_Ua dZVUg]g'ni
VY\Ui dhYb"

8]Y'6YXYi hi b['XYg'Dc_U'k YhhVYk YfVg

 $= XYi hgWXYb: i * VU```gd]Y`h'XYf'Dc_U`k YhhVYk YfV'Y]bY VYXYi hYbXY'Fc``Yž'XU'Yf'Ui WX'_`Y]bYfYb'JYfY]bYb'X]Y'A O[`]WX_Y]h V]YhYhž'[Y[Yb'ghAf_YfY'; Y[bYf'Ubni hfYhYb'i bX'g]WX'Ui Z bUh]cbU`Yf'6Ø\bY'ni 'dfAgYbh]YfYb''8Yf'8: 6!Dc_U``\Uh'gY]h'gY]bYf 9]bZØ\fi b[']a '\\\\\f'%-') 'nU\`\fY]WXYb'I bXYfXc[g'X]Y'7\UbWY [Y[YVYbž'[fc*Ufh][Y'@Y]ghi b[Yb'ni 'nY][Yb'''Gc`WXY , VYffUgWXi b[Yb'VY`YVYb'b]WXh'bi f'X]Y'K YhhVYk YfVgUha cgd\AfYž gcbXYfb'hfU[Yb'Ui WX'ni f'5hhfU_h]j]hAh'XYg': i *VU``g']b'XYb i bhYfYb'@][Yb'VY]''$

6YgcbXYfg'VY]'XYb'5a UhYi fj YfY]bYb'g]bX'X]Y'9]bbU\a Yb'Ui g Y]bYa 'Dc_U`gd]Y`'YggYbn]Y``ž'XU'g]Y'cZh'X]fY_h']b'X]Y J YfY]bgghfi _hi f'fY]bj Ygh]Yfh'k YfXYb'_ObbYb"'9]b'[i hYg 5VgW\bY]XYb'_Ubb'X]Y'Z]bUbn]Y``Y'@U[Y'XYi h`]W\'ghUV]`]g]YfYb'i bX gc[Uf'XYb'JYfY]b']b'XYf'FY[]cb'VY_UbbhYf'a UW\Yb"

Dc_U`ØVYffUgWib[Yb`XYfJYf[Ub[Yb\Y]h

K YbXi b[Yb'dUgg]YfYb'_ÒbbYbž'X]Y'XYb'A UbbgW\UZhYb'bYi Y < c ZZbi b['[YVYb"

 $\label{thm:converse} Hfc hn`XYf`UbYf_UbbhYb`GW\k YfY`XYg`VYj cfghY\YbXYb`Gd]Y`g`k]fX Yg`ZØf`XYb`<UfVi f[Yf`H6`Y]bY`; Y`Y[Yb\Y]h`gY]bž`g]W\`ni``VYk Y]gYb i bX`a <math>O[`]W\Yfk Y]gY`X]Y`: Ubg`ni``VY[Y]ghYfb"`8]Y`5bdUggi b[``Ub X]Y`; Y[YVYb\Y]hYb`i bX`XYf`i a [Y_Y\fhY`Cdh]a]ga i g`XYg HfU]bYfg`_ObbhYb`Y]bY`dcg]h]j Y`Fc``Y`gd]Y`Ybž`X]Y`]b`UbXYfYb Dc_U`gd]Y`Yb`9fZc`[`\Yfj cf[YVfUW\h`\Uh"$

Dc_U`gd]Y`Y`]a 'J Yf[`Y]W\ 'ni '@][Ugd]Y`Yb

A]h'XYf'I b[Yk]gg\Y]h'i bX'XYf'GdUbbi b[ž'X]Y'Dc_U`gd]Y`Y'a]h'g]W\Vf]b[Ybž'Zi b[]Yfh'XYf'K YhhVYk YfV'b]W\h'bi f'U`g'gdcfh`]W\Y < YfUi gZcfXYfi b[ž'gcbXYfb'Ui W\'U`g'k]W\h][Y'D`UhhZcfa 'ZØf'X]Y 5bgdfUW\Y'j cb': Ubg'i bX'X]Y'GhÀf_i b['XYf'J YfY]bg]XYbh]hÀh"

8YhU]`g

6 Y gi W Y b 'G] Y 'i b g 'U i Z . 'X] Y ! b U W f] W h Y b "b Y h