

: Ua]`]Y 'Ybh_ca a h': `Ua a Yb\Ò``Y. 'I bZU`` Ui Z'6- 'VY]'A Ø`\Y]a !? Àf`]W

: Ua]`]Y`fYhhYh`g]W\`Ui g`VfYbbYbXYa `5i hc`bUW\`I bZU```Ui Z XYf`6- "`8Yf`: U\fYf`XYg`j Yfi fgUW\YbXYb`: U\fnYi [g`ghUbX i bhYf`5`_c\c`Y]bZi gg"`*\hnh`Xfc\h`Y]b`GhfUZ| YfZU\fYb"

9]b YfgWX fYW_YbXYf'J cfZU```Ui Z'XYf'6i bXYgghfU» Y'- 'VY]'A Ø`\Y]a! ? Àf`]WX'\Uh'Ua'&) "'5i [i gh'&\$&('ZØf'5i ZfY[i b['[Ygcf[hž'U`g'Y]b 5i hc']b': `Ua a Yb'Ui Z[]b["'; `ØW_`]WXYfk Y]gY'[Y`Ub['Yg'Y]bYf ZØbZ_ÒdZ][Yb': Ua]`]Yž'g]WX 'fYWX hnY]h]['Ui g']\fYa 'VfYbbYbXYb : U\fnYi ['ni 'fYhhYb"'8]Y'G]hi Uh]cb'ZØ\fhY'b]WX h'bi f'ni 'Yf\YV`]WXYb J Yf_Y\fgVY\]bXYfi b[Ybž'gcbXYfb'k Uf'Ui WX'Y]b'K YW_fi Z \]bg]WX h`]WX 'XYf'; YZU\fYbž'X]Y'5`_c\c``Ua 'GhYi Yf'a]h'g]WX Vf]b[Yb'_Ubb"

8 Yf'I b ZU``YfY][bYhY'g]W\ž'U`g'Y]b'b UW\Zc`[YbXYg'5i hc'Ui Z'XUg : Ua]`]Yb ZU\fnYi ['Ui ZZi \f"'8]Y'[YbUi Y'I fgUW\Y'XYg'5i ZdfU``g']gh bcW\ib_`Ufž'XcW\'X]Y'?cbgYei YbnYb'k UfYb'XfUa Uh]gW\"'8 Ug 5i hc'XYf': Ua]`]Y'dfU``hY'[Y[Yb'X]Y'@Y]hd`Ub_Yž'k Ug'ni f 9bhnØbXi b['XYg': U\fnYi [g'ZØ\fhY"'Gc`W\Y'I bZÀ``Y'nY][Yb'Ui Z YfgW\fYW_YbXY'K Y]gYž'k]Y'gW\bY``'Ui g'Y]bYa 'U``hÀ[`]W\Yb : U\fj Yf`Ui Z'Y]bY'dchYbn]Y````YVYbgVYXfc\`]W\Y'G]hi Uh]cb'k YfXYb _Ubb"

8YhU]`g`XYg`JcfZU``g

8]Y: Ua]`]Yž'VYghY\YbX'Ui g'ZØbZ'DYfgcbYbž'_cbbhY'g]W\gY`Vgh'Ui g XYa '5i hc'VYZfY]Ybž'VYj cf'X]Y: `Ua a Yb'XUg': U\fnYi ['j c``ghÀbX][Yf[f]ZZYb"'9g']gh'Y]b'; `ØW_gZU``ž'XUgg'b]Ya UbX'gW\k YfYf'j Yf`Yhnh k i fXYž'UV[YgY\Yb'j cb'Y]bYa '?]bXž'XUg'a]h'?cdZj Yf`Yhni b[Yb']a ?fUb_Yb\Ui g'VY\UbXY`h'k YfXYb'a i gghY"'8]Y'UbXYfYb : Ua]`]Yba]h[`]YXYf`Yf`]hhYb'[` \emptyset W_`]WXYfk Y]gY'bi f'`Y]WXhY J Yf`Yhni b[Ybž'k Ug'X]Y'G]hi Uh]cb'Yhk Ug'a]`XYfh"

K]Y 'g]W '\YfUi gghY ``hYžghUbX 'XYf'a i ha U» `]W Y 'J Yfi fgUW Yf'XYg I bZU``g'i bhYf'Yf\YV`]W Ya '5`_c\c`Y]bZi gg"'8]Yg'ghY``hY 'Y]bYb k Y]hYfYb '5b[Ub[gdi b_h'ZØf'X]Y 'Y]b[YgYhnhYb 'Dc`]nY]VYUa hYb 'XUfž X]Y ']\a 'b]W h'bi f'X]Y : U\fYf`Ui Vb]g'Ybhnc[Ybž'gcbXYfb'Ui W 'Y]b GhfUZJ YfZU\fYb'[Y[Yb']\b'Y]b`Y]hYhYb"'Gc`W Y 'A U» bU\a Yb'g]bX bchk YbX][ži a 'X]Y 'G]W Yf\Y]h'Ui Z'i bgYfYb'GhfU» Yb'ni [Yk À\f`Y]ghYb'i bX'ni 'j Yf\]bXYfbž'XUgg'g]W 'À\b`]W Y'J cfZÀ``Y k]YXYf\c`Yb"

8]Y'5i gk]f_i b[Yb'XYg'I bZU``g'k UfYb'b]Wh'bi f'Ui Z'X]Y'X]fY_h 6YhY]`][hYb'VYgWkfÀb_h"'8]Y'6i bXYgghfU»Y'- 'k i fXY'ZØf'a Y\fYfY Ghi bXYb'[YgdYffhž'k Ug'ni '[fc»Yb'JYf_Y\fgVY\]bXYfi b[Yb'i bX K UfhYnY]hYb'ZØf'j]Y'Y'DYbX`Yf'ZØ\fhY"'8]Y'Dc`]nY]'i bX': Yi Yfk Y\f a i gghYb'fUgWk'Y]b[fY]ZYbž'i a 'gck c\``XYb'6fUbX'ni '`ÒgWkYb'U`g Ui Wk'XYb'JYf_Y\f'i a ni `Yb_Yfb"'Gc`WkY'GnYbUf]Yb'j YfXYi h`]WkYb X]Y'8f]b[`]Wk_Y]hY]bYg'j YfUbhk cfhi b[gVYk i gghYb : U\fj Yf\U`hYbg"

8 Ug'[fc»Y'; UbnY

±0.5 b V Y h f U W \ h ' X Y f'; Y Z U \ f Y b ž' X] Y 'a] h '5 `_c \ c `` U a ``GhY i Y f j Y f V i b X Y b 'g] b X ž'] g h 'Y g 'k] W \ h] [ž' X U g g : U \ f Y f 'g h Y h g j Y f U b h k c f h i b [g V Y k i g g h \ U b X Y `b " `GhU h] g h] _ Y b 'n Y] [Y b ž' X U g g U `_c \ c `V Y X] b [h Y `I b Z A ``Y 'Y] b Y `X Y f \ A i Z] [g h Y b `I f g U W Y b `Z Ø f J Y f _Y \ f g h c h Y 'g] b X " 8] Y g Y g '9 f Y] [b] g _ O b b h Y `X] Y `8] g _ i g g] c b 'Ø V Y f X] Y `B c h k Y b X] [_Y] h '] b h Y b g] j Y f Y f `G] W Y f \ Y] h g a U » b U \ a Y b 'i b X 5 i Z _`A f i b [g _ U a d U [b Y b '] a `Gh f U » Y b j Y f _Y \ f `U b \ Y] n Y b "

Ni XYa []Vh'Yg']a a Yf'k]YXYf'5ddY``Yž'X]Y'; YgY``gW\UZh'ni gYbg]V]`]g]YfYb'ZØf'X]Y'; YZU\fYb'j cb'5`_c\c``gck]Y'X]Y

J YfUbhk cfhi b[ž'X]Y'YXYf': U\fYf'hfÀ[h''8Yf': U```j cb'A Ø`\Y]a!
?Àf`]W\'gc``hY'U`g'Y]bXf]b[`]W\Y'9f]bbYfi b['X]YbYbž'k]Y'k]W\h]['Yg]ghž'bØW\hYfb'i bX'Ui Za Yf_gUa'\]bhYf'XYa 'GhYi Yf'ni 'g]hnYb''8]Y
?cbgYei YbnYb'j cb': Y\`j Yf\U`hYb'[Y`hYb'b]W\h'bi f'ZØf'XYb'HÀhYfž

9]b'k Y]hYfYf'5gdY_h']gh'X]Y'Fc``Y'XYf'FYhhi b[gX]YbghYž'X]Y'gW\bY``
j cf'Cfh'k UfYb"'=\fY'gW\bY``Y'FYU_h]cb'\Uh'a Ò[`]W\Yfk Y]gY'XUni
VY][YhfU[Ybž'[fÒ» YfY'GW\ÀXYb'ni 'j Yf\]bXYfb'i bX'X]Y': Ua]`]Y
nØ[]['Ui g'XYf'; YZU\fYbncbY'ni 'Vf]b[Yb"'Gc`W\Y'9]bgÀhnY
YfZcfXYfb'b]W\h'bi f'A i hž'gcbXYfb'Ui W\'\c\Y': UW_ca dYhYbn']a
I a [Ub['a]h'_f]h]gW\Yb'G]hi Uh]cbYb"

±g[YgUa hinY][hiX]YgYfiJcfZU``žik]YijYf`Yhn`]WXik]fi]a GhfU»YbjYf_Y\fig]bXiibXik]YigWXbY``ig]WXiXUgi@YVYbiUiZiXYbi?cdZghY``Ybi_Ubb"i≯XYfigc``hYig]WXiXYggYbiVYkiigghigY]bžikÀ\fYbXiYf\]bhYfiXYa iGhYiYfig]hnhiibXiX]YiJYfUbhkcfhib[iZØfigY]biY][YbYg@YVYbiibXiXUgiUbXYfYfihfÀ[h"

I fgUWX Yb i bX :: c`[Yb j cb J Yf_Y\fgi bZÀ``Yb

 $\label{thm:continuity:continuity:bold} J Yf_Y \ fgi b Z \ YY \ g b X Y \ b Y \ fgi b Z \ YY \ Y \ g b Y \ fgi b Z \ YY \ YY \ g b Y \ fgi b Z \ YY \ YY \ g b Y \ fgi b Z \ YY \ YY \ g g \ g b Y \ fgi b Z \ YY \ fgi b Z \ fhy b Z \ fhy$

 $8]Y : c`[Yb : gc`W.Yf'I bZA``Y : g]bX : j]Y`ZA`h][""BYVYb : XYb : d \ mg]gW.Yb J Yf`Yhni b[Yb : i bX : XYa : Ya ch]cbU`Yb : HfUi a Už: XUg: 6Yhfc ZZYbY : i bX]\ fY: 5b[Y\Of][Yb : Yf`YVYb ž: Yb hghY\Yb : Ui : W. : Yf\YV`]W.Y [YgY``gW. UZh`]W.Y : i bX : k]fhgW. UZh`]W.Y :? cghYb : "=a : *J\f'.&$.&% k : fXYb : X]Y : k]fhgW. UZh`]W.Yb :? cghYb : Xi : fW. : J Yf_Y\fgi : bZA``Y :]b 8 Yi : hgW. `UbX : Ui : Z'Yhk : U'' : "A]``]UfXYb : 9i : fc : [YgW.Ahnh": 8 Uni : nA\`Yb : b]W. h'bi : f'X]Y : i : ba]hhY`VUfYb : a : YX]n]b]gW.Yb :? cghYb ž: gcb XYfb : Ui : W.X]Y :`Ub[Zf]gh][Y:FY\UV]`]hUh]cb : i : bX:X]Y : J Yf`i : ghY:Xi : fW. 5 fVY]hgi : bZA\][_Y]h"$

; YgYhn`]WXYAU»bU\a Yb ni f

$JYf_Y\fgg]WXYfYh$

I a 'X]Y'JYf_Y\fgg]W\Yf\Y]h'ni 'Yf\O\Ybž\Uh'8Yi hgW\`UbX'a Y\fYfY [YgYhn`]W\Y'A U» bU\a Yb']a d`Ya Ybh]Yfh"'8Uni 'nÀ\`Yb'ghfYb[YfY JcfgW\f]ZhYb'ZØf': U\fnYi []bgdY_h]cbYbž'XYf'9]bgUhn'j cb; YgW\k]bX][_Y]hga Ygg[YfÀhYb'i bX'X]Y', VYfk UW\i b['XYg 5`_c\c`[fYbnk YfhYg'Ua 'GhYi Yf"'8]Y'Dfca]``Y[fYbnY'ZØf : U\fUbZ\hz[Yf'i bX'6Yfi Zg_fUZhZU\fYf'`]Y[h'VY]'\$ž\$ž'k \ha\fYbX'ZØf U``Y'UbXYfYb': U\fYf'Y]b'; fYbnk Yfhj cb'\$ž)'[]`h"'6Y]'JYf[Y\Ybž'X]Y i bhYf'5`_c\c`Y]bZi gg'VY[Ub[Yb'k i fXYbž'_ObbYb'Ya dZ]bX`]W\Y; Y`XghfUZYb'i bX': U\fYfVchY'j Yf\\hab[h'k YfXYb"

Ni gàhn`]W\ni `XYb`[YgYhn`]W\Yb`JcfgW\f]ZhYb`k]fX`Ui W\j Yfghàf_h]b`5i Z_`àfi b[g_Ua dU[bYb`]bj Ygh]Yfhži a `XUg`6Yk i gghgY]b`ZØf`X]Y; YZU\fYb`j cb`5`_c\c``Ua `GhYi Yf`ni `gW\àfZYb"`Cf[Ub]gUh]cbYbk]Y`X]Y`8Yi hgW\Y`JYf_Y\fgk UW\h`]bhYbg]j]YfYb`]\fY6Ya Ø\i b[Ybži a `Y]bY`g]W\YfYfY`JYf_Y\fg]bZfUghfi _hi f`niZÒfXYfb`i bX`dfàj Ybh]j Y`A U»bU\a Yb`ni `i bhYfghØhnYbž`X]Y`\Y`ZYbž X]Y`NU\``XYf`JYf_Y\fgi bZà``Y`ni `j Yff]b[Yfb"

9] $bZiggicb5^ccciUiZXUg:U\fjYf\UhYb$

5`_c\c`_cbgi a `\Uh'k Y]hfY]W\YbXY'5i gk]f_i b[Yb'Ui Z'X]Y
: À\][_Y]h'XYg': U\fYfgž'g]W\Yf'ni 'ZU\fYb"'6YfY]hg'VY]'Y]bYa
6`i hU`_c\c`k Yfh'j cb'\$ž' 'Dfca]``Y'_Ubb'X]Y': U\fZÀ\][_Y]h
VYY]bhfÀW\h][h'k YfXYbž'k Ug'ni 'Y]bYf'Yf\Ò\hYb'I bZU``[YZU\f'ZØ\fh"
5`_c\c`'j Yf`Ub[gUa h'X]Y'FYU_h]cbgnY]hYbž'VYY]bhfÀW\h][h'XUg
I fhY]`gj Yfa Ò[Yb'i bX'j Yff]b[Yfh'X]Y'K U\fbY\a i b['j cb'; YZU\fYb
]a 'GhfU» Ybj Yf_Y\f"

@Ui h'Y]bYf'Ghi X]Y'XYg'JYf_Y\fgWi Vg'8Yi hgW\`UbX'fJ78Ł'g]bX fi bX'' \$'DfcnYbh'U``Yf'hÒX`]W\Yb'JYf_Y\fgi bZÀ``Y']b'8Yi hgW\`UbX Ui Z'5`_c\c``ni fØW_ni ZØ\fYb"'8]YgY'U`Ufa]YfYbXYb'NU\`Yb i bhYfghfY]W\Yb'X]Y'K]W\h][_Y]h'j cb'5i Z_`Àfi b['i bX'DfÀj Ybh]cb']b 6Yni ['Ui Z'XYb'5`_c\c`_cbgi a 'i bX'XUg': U\fYb"

: Øf`k Y]hYfY`±bZcfa Uh]cbYb`ni f`J Yf_Y\fgg]W\Yf\Y]h`i bX`XYb`XUa]h j YfVi bXYbYb`H\Ya Ybž`VYgi W\Yb`G]Y`X]Y`K YVg]hY`XYg`

8 Y hU]`c

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'X]Y!bUWX f]WX hYb "b Yh